# Mono-monostatic bodies
# UNDER CONSTRUCTION

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A *convex* body resting on a plane is in static equilibrium if the distance of the boundary of the body from its barycenter is stationary at the point of contact between the body and the plane. Such equilibrium is stable if the distance is a local minimum, and unstable otherwise.

A body is called *monostatic* if has only one stable configuration, and *mono-monostatic* if it has one stable and one unstable configuration. Since the boundary of the body is compact, and since the distance from the barycenter is a a continuous function, every body must have at least one stable and one unstable equilibrium configuration.

Although one can construct several examples of monostatic bodies, the task is less trivial if, in addition, the body is required to be homogeneous, since in that case the position of the barycenter and the shape of the body are not independent. Indeed, in 1994 Domokos, Papadopoulos and Ruina published a paper on the Journal of Elasticity showing that in two dimensions a *homogeneous* body must have at least four equilibria, and hence at least two stable equilibria. Thus, **homogeneous monostatic bodies cannot exist in two dimensions**.

In three-dimensions one can construct examples of homogeneous bodies with have only one stable equilibrium. An example is a cylinder chopped at a right angle on one end and at a skew angle at the other end. However, **one may ask whether the minimality of the number of four equilibria (stable or unstable) extends to the three dimensional setting.** Vàrkonyi and Domokos have shown that this is not the case by constructing an example of a mono-monostatic body. In a paper published in 2006 on the Journal of Nonlinear Science they provide an explicit construction of a mono-monostatic body.

They defining the surface of such body through a distance function which prescribes, in a system of polar coordinates, the radius as a function

$$\varrho = R(\theta, \varphi). \tag{1}$$

To arrive at this function, they consider a wider class of functions of the form

$$R(\theta, \varphi; c, d) = 1 + d\Delta R(\theta, \varphi, c) \tag{2}$$

which depend on the parameters $c$ and $d$, with the latter tuning the departure of this shape from a sphere. For $c \to \infty$, the shape described by this function is an egg... to complete