1 Growth and stability

1.1 Growth-induce elastic instability

Growth of an elastic body can induce mechanical instability. As an example, consider an Euler beam constrained between two simple supports placed apart at distance ℓ_0. Suppose that the beam can undergo growth, in the sense that its natural length ℓ can change. The compressive force in the beam is

$$P = EA(\ell_0 - \ell). \tag{1}$$

Accordingly, the beam buckles when the length reaches the critical value

$$\ell_{\text{crit}} = \ell_0 + \pi^2 \frac{EI}{EA\ell_0^2}. \tag{2}$$

1.2 Unstable growth

A more interesting instability phenomenon can take place when the evolution law that governs the length of the beam depends on the stress. This type of instability has been discussed in the paper [1] in a broader context, inspired by experiments on actin growth.

The mechanical system studied in [1] consists of a one-dimensional elastic bar shown clamped on one side and constrained at the other side by an soft device, as shown in the figure. The name bar instead of beam means that this object

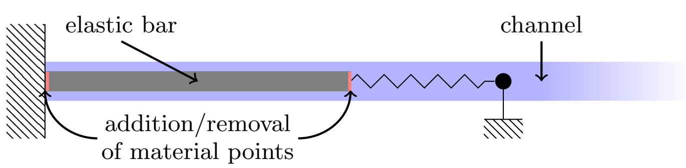

Figure 1: An elastic bar clamped between a hard and a soft device, immersed in a semi-infinite channel.

can stretch but cannot bend, hence its motion takes place in a one-dimensional physical space. For this reason, the bar cannot buckle. Nevertheless, the system can undergo instabilities because of the coupling between growth and stress state. The main point of this work is to discuss how the interplay between growth and stress is responsible for instability.

To incorporate growth in the model, we assume that the physical space contains a chemical species which can freely diffuse, and whose elements can attach and detach to and from the bar, only at its extreme points.

The key ingredient for the description of growth is a kinetic equation of the form

$$V = f(\sigma, \mu), \tag{3}$$

- A discussion on the result of the paper can be found here.
- Various notes and questions about the paper can be found here

2 References

1. Rohan Abeyaratne, Eric Puntel, and Giuseppe Tomassetti. Treadmill stability of a one-dimensional actin growth model. *In preparation*, 2019.