April 18, 2019

Question: The question is about the relation between the concentration of a diffusant and the chemical potential.

In a one-dimensional (actually the question is for any dimension) steady flow of a diffusant over a interval $[y_0(t), y_1(t)]$ we have the flux $h(y,t)$ and the chemical potential as $\mu(y,t)$ obeying

$$h(y,t) = -D\frac{\partial \mu}{\partial y}, \qquad \frac{\partial h}{\partial y} = 0 \tag{1}$$

we obtain

$$\mu(y,t) = \mu_1(t)\frac{y - y_0(t)}{y_1(t) - y_0(t)} + \mu_0(t)\frac{y_1(t) - y}{y_1(t) - y_0(t)}, \tag{2}$$

and so

$$h(y,t) = -D\,\frac{\mu_1(t) - \mu_0(t)}{y_1(t) - y_0(t)} \tag{3}$$

If instead I work in terms of the diffusant concentration $c(y,t)$ we might write Fick's law as

$$h(y,t) = -\kappa\frac{\partial c}{\partial y} \tag{4}$$

which together with $\partial h/\partial y = 0$ leads to

$$c(y,t) = c_1(t)\frac{y - y_0(t)}{y_1(t) - y_0(t)} - c_0(t)\frac{y - y_1(t)}{y_1(t) - y_0(t)}. \tag{5}$$

and so

$$h(y,t) = -\kappa\,\frac{c_1(t) - c_0(t)}{y_1(t) - y_0(t)} \tag{6}$$

Can I compare (3) with (6) and conclude that

$$\mu_1 = c_1\kappa/D, \qquad \mu_0 = c_0\kappa/D\ ? \tag{7}$$

More generally is

$$\mu(y,t) = c(y,t)\kappa/D\ ? \tag{8}$$

What I would like to understand is whether we can write the kinetic laws

$$-\dot{x}_0 = \frac{\rho(\mu_0 - \overline{\mu}_0) + \overset{*}{W}(\sigma_0)}{B_0} = \frac{\rho\kappa}{DB_0}c_0 - \frac{\rho\overline{\mu}_0}{B_0} + \frac{\overset{*}{W}(\sigma_0)}{B_0} = k^0_{on}c_0 - k^0_{off} \tag{9}$$

$$\dot{x}_1 = \frac{\rho(\mu_1 - \overline{\mu}_1) + \overset{*}{W}(\sigma_1)}{B_1} = \frac{\rho\kappa}{DB_1}c_1 - \frac{\rho\overline{\mu}_1}{B_1} + \frac{\overset{*}{W}(\sigma_1)}{B_1} = k^1_{on}c_1 - k^1_{off} \tag{10}$$

where k_{off} would be a function of stress. The biologists like to write there growth laws in this latter way.