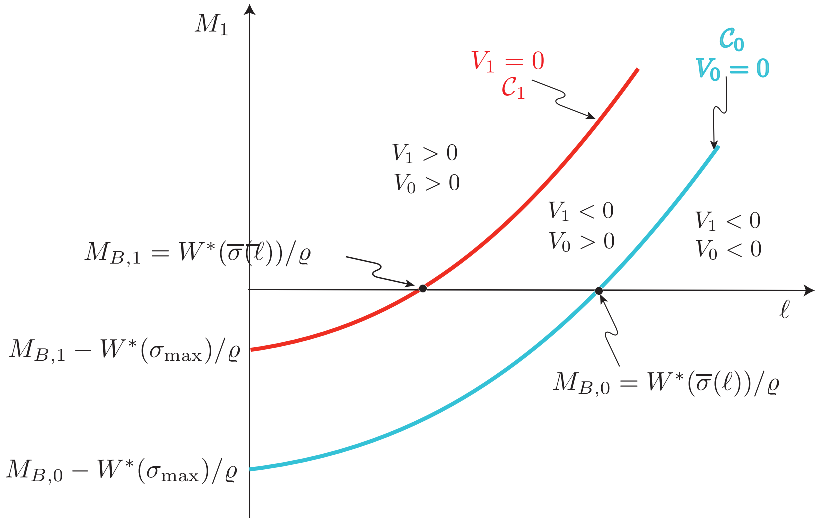

Figure 1: The ℓ, M_1-plane in the case $M_{B,1} > M_{B,0}$. The growth rates V_0 and V_1 vanish on the respective curves $\mathcal{C}_0$ and $\mathcal{C}_1$. These curves demarcate the ℓ, M_1-plane into regions where the signs of the growth velocities are as shown

It is informative to examine how each end of the bar grows in different regions of the ℓ, M_1-plane. The curves $\mathcal{C}_0$ and $\mathcal{C}_1$ at which $V_0 = 0$ and $V_1 = 0$ are shown in Figure 1 in the case $M_{B,1} > M_{B,0}$ that is of principal interest. By (**??**) (32) and (**??**) (29) they are characterized by

$$\mathcal{C}_0: \ \ M_1 = M_{B,0} - W^*(\overline{\sigma}(\ell))/\varrho, \qquad \mathcal{C}_1: \ \ M_1 = M_{B,1} - W^*(\overline{\sigma}(\ell))/\varrho. \tag{1}$$

It follows from the properties of $\overline{\sigma}(\ell)$ and $W^*(\sigma)$ that $-W^*(\overline{\sigma}(\ell))$ increases monotonically from $W^*(\sigma_{\max})$ to $+\infty$ as ℓ increases from $\ell = 0$. These curves demarcate the ℓ, M_1-plane into 3 regions where the signs of the growth velocities $V_0 = -\dot{x}_0$ and $V_1 = \dot{x}_1$ are as shown. Observe, for example, that at any fixed value of the chemical potential M_1 exceeding $M_{B,1} - W^*(\sigma_{\max})/\varrho$, if the bar is sufficiently long, specifically to the right of $\mathcal{C}_0$, ablation happens at both ends ($V_1 < 0, V_0 < 0$) and the bar will grow shorter. On the other hand on the left of $\mathcal{C}_1$, where the bar is sufficiently short, accretion happens at both ends ($V_1 > 0, V_0 > 0$) and the bar will grow longer. Between the two curves $\mathcal{C}_0$ and $\mathcal{C}_1$, where the bar has an intermediate length, accretion occurs at the left-hand end ($V_0 > 0$) and ablation occurs at the right-hand end ($V_1 < 0$).

The curve $\mathcal{C}_{TM}$ corresponding to treadmilling is found by setting $V_0 = -V_1$, i.e. $\dot{x}_0 = \dot{x}_1$, which from (**??**) (32) is found to be described by

$$\mathcal{C}_{TM}: \qquad M_1 = \frac{M_{B,0} + \beta\left[1 + \frac{\rho^2}{mB_0K}(\sigma_{\max} - \overline{\sigma}(\ell))\right] M_{B,1}}{1 + \beta\left[1 + \frac{\rho^2}{mB_0K}(\sigma_{\max} - \overline{\sigma}(\ell))\right]} - W^*(\overline{\sigma}(\ell))/\varrho. \tag{2}$$

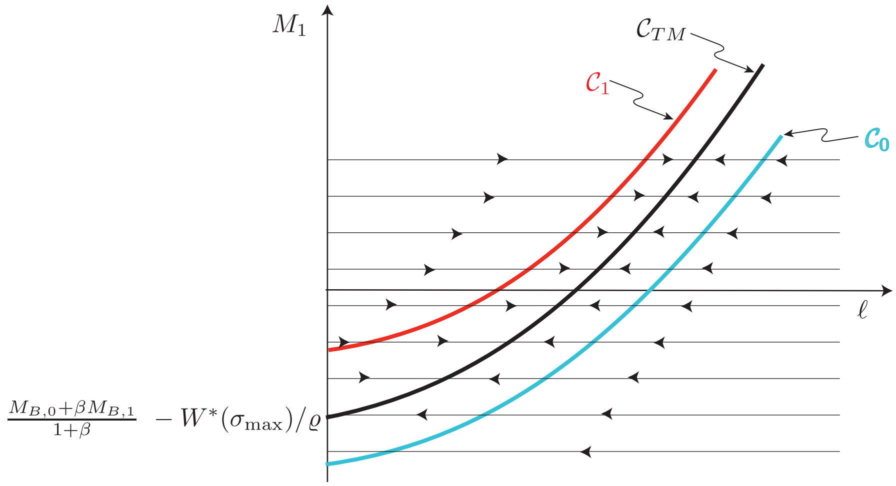

Figure 2: A point (ℓ, M_1) on the curve $\mathcal{C}_{TM}$ corresponds to a treadmilling state. The arrow at an arbitrary point (ℓ, M_1) tells us whether $\dot{\ell}$ is positive or negative there thus indicating whether the length of the bar increases or decreases. Observe from the figure that a treadmilling solution exists whenever (3) holds.

It is clear from (1) and (2) that $\mathcal{C}_{TM}$ lies between the two curves $\mathcal{C}_0$ and $\mathcal{C}_1$ and this is shown in Figure 2. By setting $\ell = 0$ in (2) we see that $\mathcal{C}_{TM}$ cuts the M_1-axis at

$$M_1 = \frac{M_{B,0} + \beta M_{B,1}}{1+\beta} - W^*(\sigma_{\max})/\varrho.$$

It follows that, corresponding to any given value of the chemical potential

$$M_1 > \frac{M_{B,0} + \beta M_{B,1}}{1+\beta} - W^*(\sigma_{\max})/\varrho, \tag{3}$$

there is a corresponding length of the bar $\ell = \ell_{TM}$ such that (ℓ_{TM}, M_1) lies on $\mathcal{C}_{TM}$, or stated differently, a unique treadmilling solution exists whenever (3) holds. It is not difficult to show that the inequality (3) is equivalent to the inequality (**??**) (42) (which in turn is equivalent to (**??**) (40)).

If the chemical potential of the free monomers does not obey (3), i.e. if

$$M_1 < \frac{M_{B,0} + \beta M_{B,1}}{1+\beta} - W^*(\sigma_{\max})/\varrho, \tag{4}$$

we see from Figure 2 that (a) there is no corresponding treadmilling state, and (b) if the bar has some positive length at the initial instant, it will mononotically get shorter and eventually disappear.

Now consider the **case** $M_{B,1} < M_{B,0}$. Figure 3 shows the ℓ, M_1-plane and the signs of the growth velocities on different regions of it. INCOMPLETE

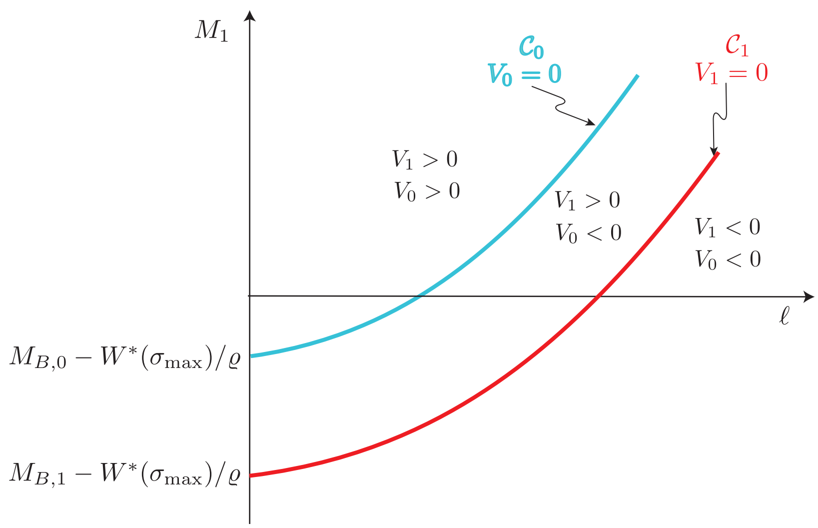

Figure 3: The ℓ, M_1-plane in the case $M_{B,1} < M_{B,0}$. The growth rates V_0 and V_1 vanish on the respective curves $\mathcal{C}_0$ and $\mathcal{C}_1$. These curves demarcate the ℓ, M_1-plane into regions where the signs of the growth velocities are as shown