A nonlinear theory for fibre-reinforced magneto-elastic rods

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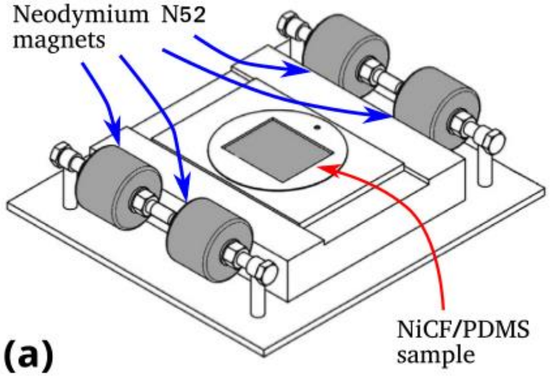
Based on the joint work with J. Ciambella and A. Favata (Sapienza University)



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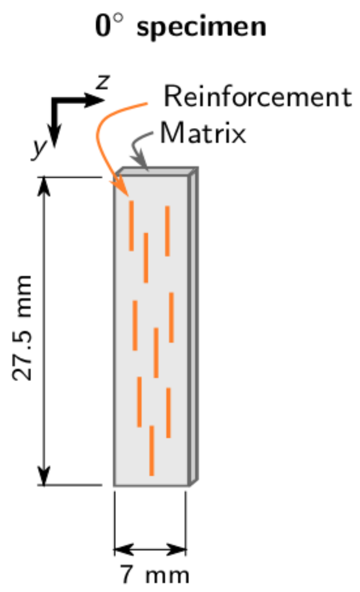

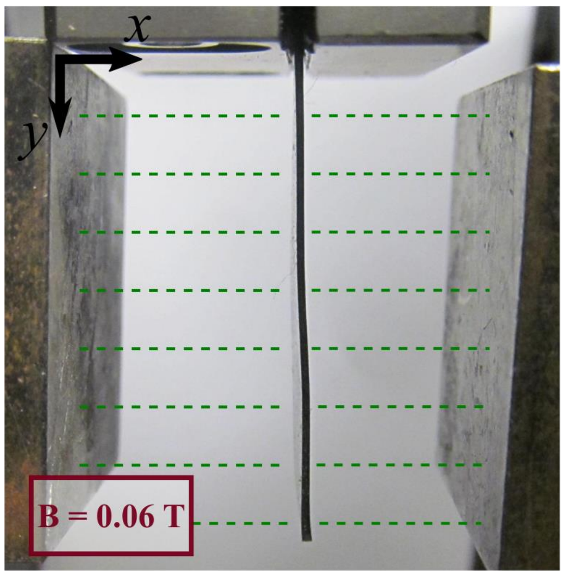

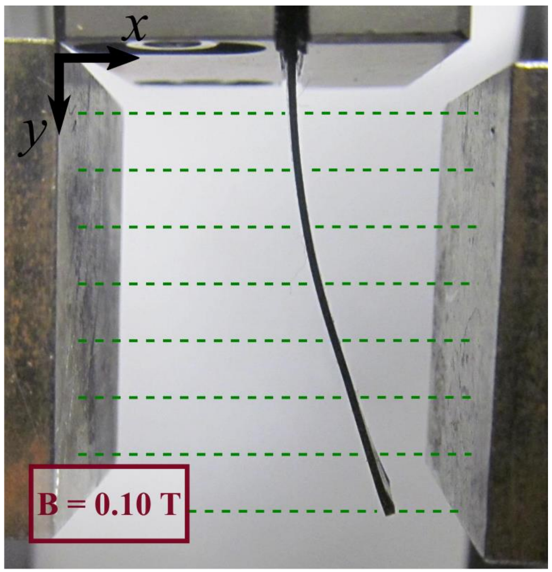

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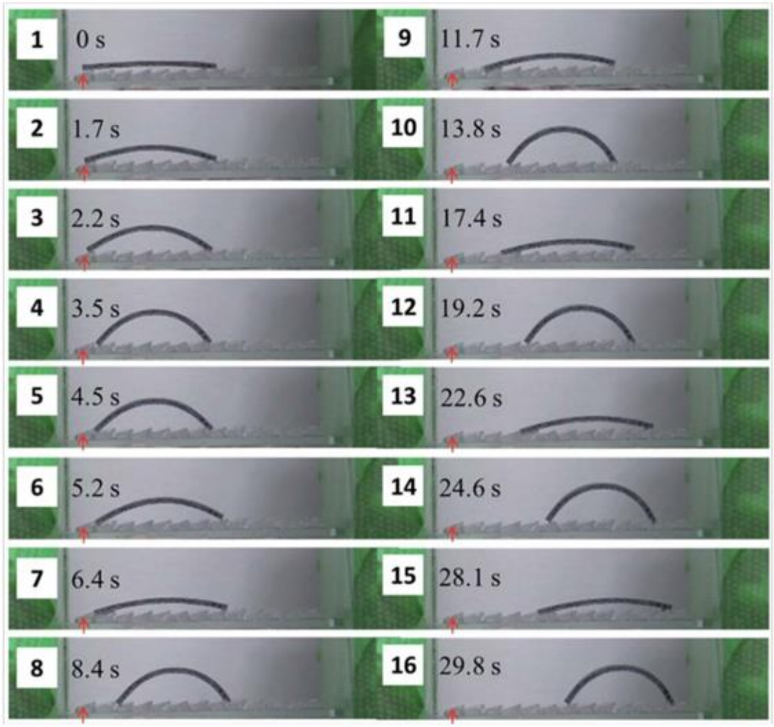

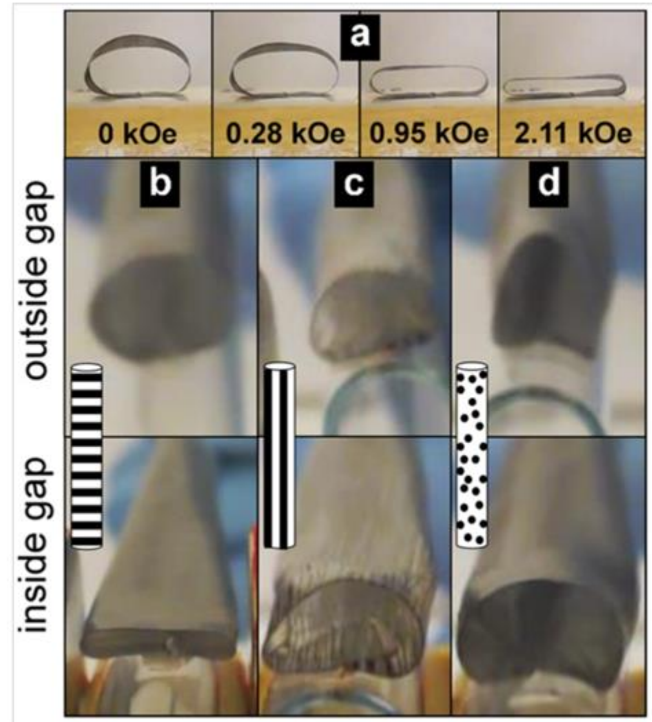

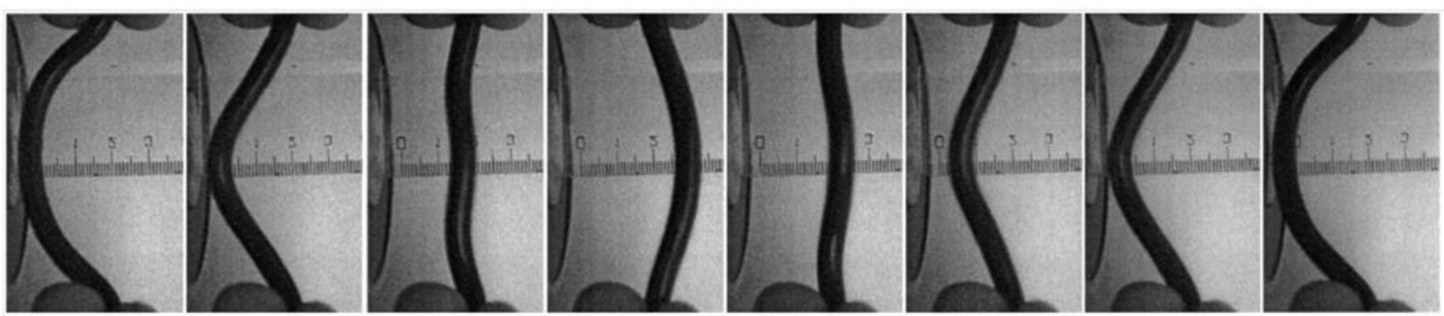

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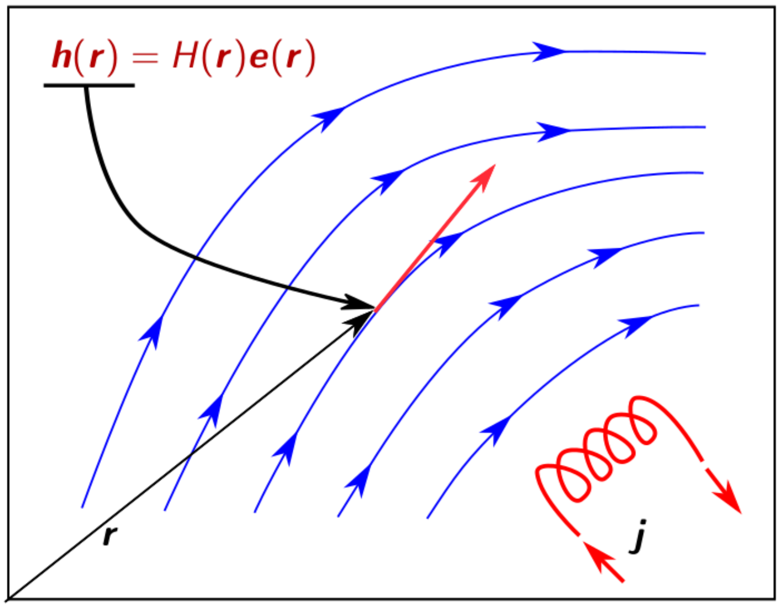
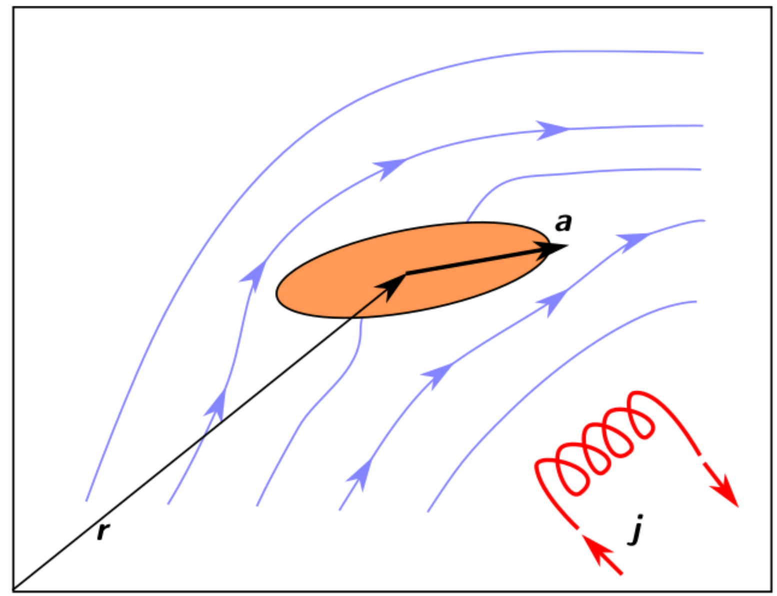
The energy of a single particle in free space





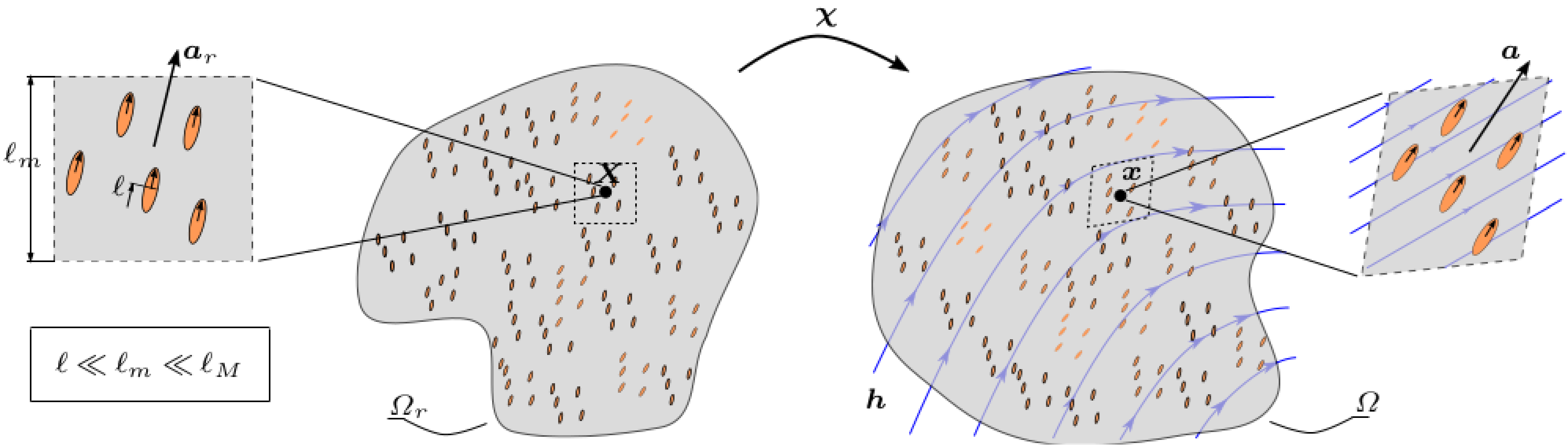
$$\mathscr{E}(\boldsymbol{r}, \boldsymbol{a}) = -\frac{\mu_0}{2}\chi H^2(\boldsymbol{r})\Big(1 + \alpha(\boldsymbol{a} \cdot \boldsymbol{e}(\boldsymbol{r}))^2\Big)$$

χ effective magnetic susceptibility

α anisotropy factor

μ_0 permeability of vacuum

The macroscopic energy



- $\boldsymbol{a}_r(X)$ orientation of a fiber at X in the reference configuration
- in the deformed configuration, the orientation of the same fiber is

$$\boldsymbol{a}(X) = \frac{\boldsymbol{F}(X)\boldsymbol{a}(X)}{|\boldsymbol{F}(X)\boldsymbol{a}(X)|} \qquad \boldsymbol{F} = \nabla\chi$$

- Macroscopic energy

$$\mathcal{E}(\boldsymbol{\chi}) = \int_{\Omega_r} W(X, \boldsymbol{F}(X)) + L(X, \chi(X), \boldsymbol{F}(X))\, \mathrm{d}V(X),$$

The macroscopic energy (cont.d)

- Macroscopic energy

$$\mathcal{E}(\boldsymbol{\chi}) = \int_{\Omega_r} W(X, \boldsymbol{F}(X)) + L(X, \chi(X), \boldsymbol{F}(X)) \,\mathrm{d}V(X),$$

- Magnetic force and magnetic stress

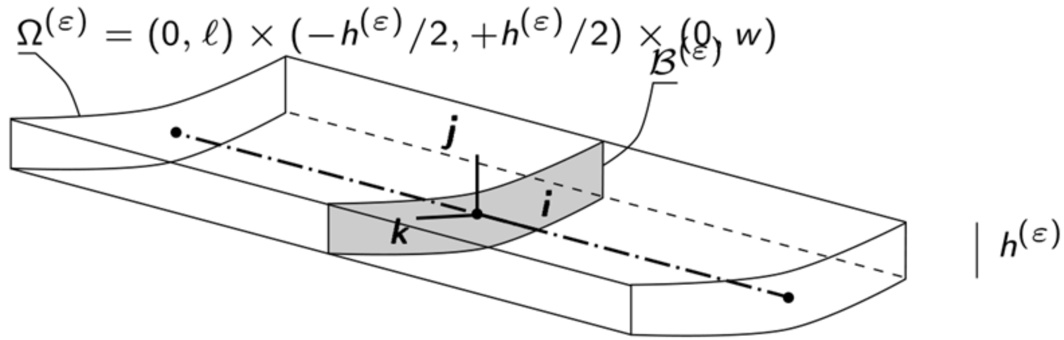
$$\boldsymbol{f}_{mag} = \partial_x L(X, x, \mathbf{F}), \qquad \boldsymbol{S}_{mag} = \partial_{\mathbf{F}} L(X, x, \mathbf{F})$$

- For **r** a translation and **R** a rotation, we have, in general

$$L(X, x, \mathbf{F}) \neq L(X, x + \mathbf{r}, \mathbf{RF}).$$

for some translation **r** and for some rotation **R**

Magneto-elastic beam



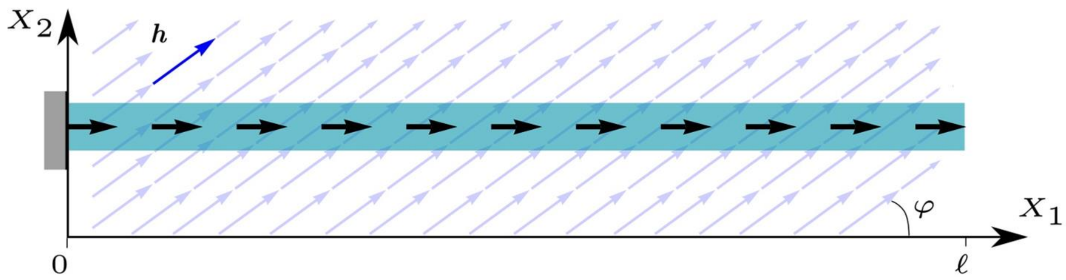
- thickness scales as $h^{(\varepsilon)} = \varepsilon h$
- the magnetic field scales as $H^{(\varepsilon)} = \varepsilon H$
- the orientation of the magnetic fibers depends only on X_1, i.e., $\boldsymbol{a} = \widehat{\boldsymbol{a}}(X_1)$
- Ansatz:

$$\boldsymbol{f}(X_1, X_2, X_3) = \boldsymbol{r}(X_1) + X_2\boldsymbol{d}(X_1), \qquad s.t. \quad |\boldsymbol{r}'| = 1, \quad \boldsymbol{d} = \boldsymbol{k}\times\boldsymbol{r}'$$

- retaining terms up to order $O(\varepsilon^2)$, we obtain

$$\mathscr{E}_{1D}(\boldsymbol{r}) = \int_0^\ell w(X_1, \boldsymbol{r}''(X_1)) + I(X_1, \boldsymbol{r}(X_1), \boldsymbol{r}'(X_1))dX_1$$

Cantilever



For a cantilever beam under uniform magnetic field the renormalized energy is

$$\widehat{\mathcal{E}}(\vartheta) = \frac{1}{2}\int_0^1 \left((\vartheta'(s))^2 - h^2(\cos(\vartheta(s)-\varphi))^2\right) \mathrm{d}s$$

where

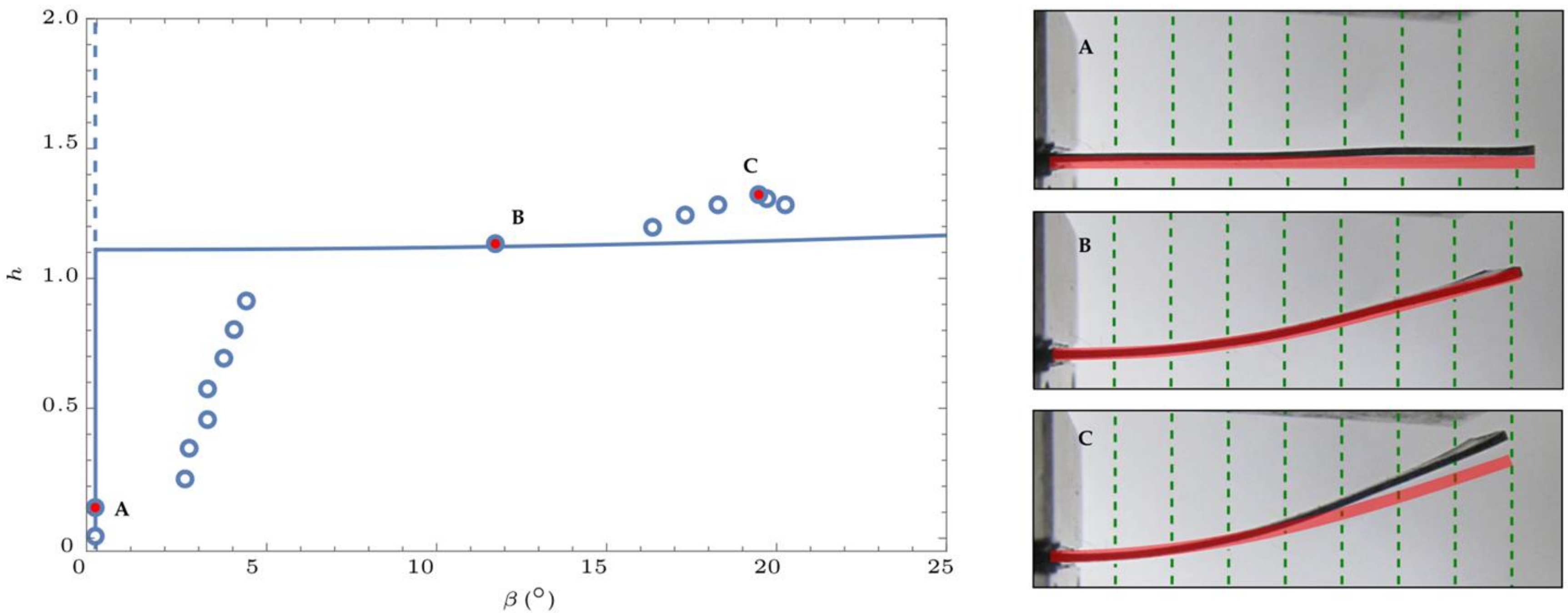
- ϑ is the inclination of $\boldsymbol{r}'$
- φ is inclination of the applied field
- h is the renormalized intensity of the applied field

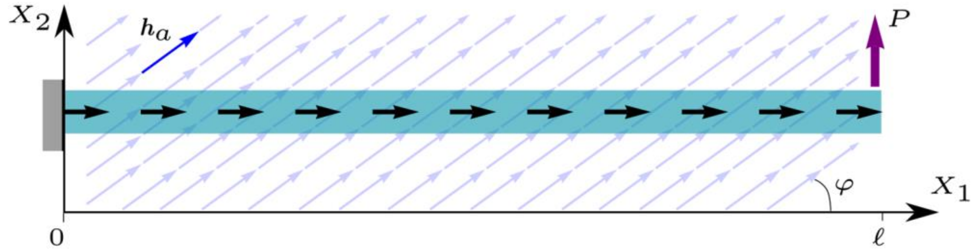
The Euler-Lagrange equation is:

$$-2\vartheta'' - h^2 \sin(2(\vartheta - \varphi))) = 0.$$

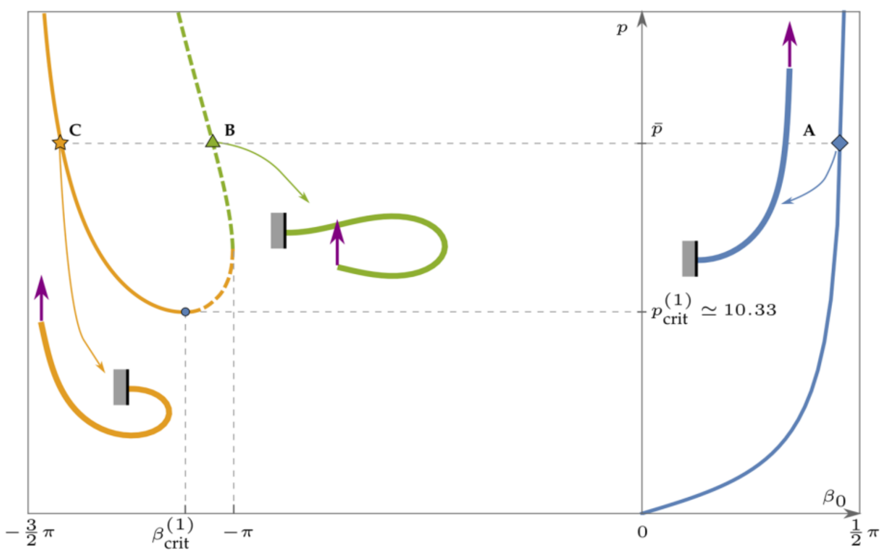
In particular, if $\varphi = \pi/2$ then

$$-2\vartheta'' + h^2 \sin(2\vartheta) = 0.$$

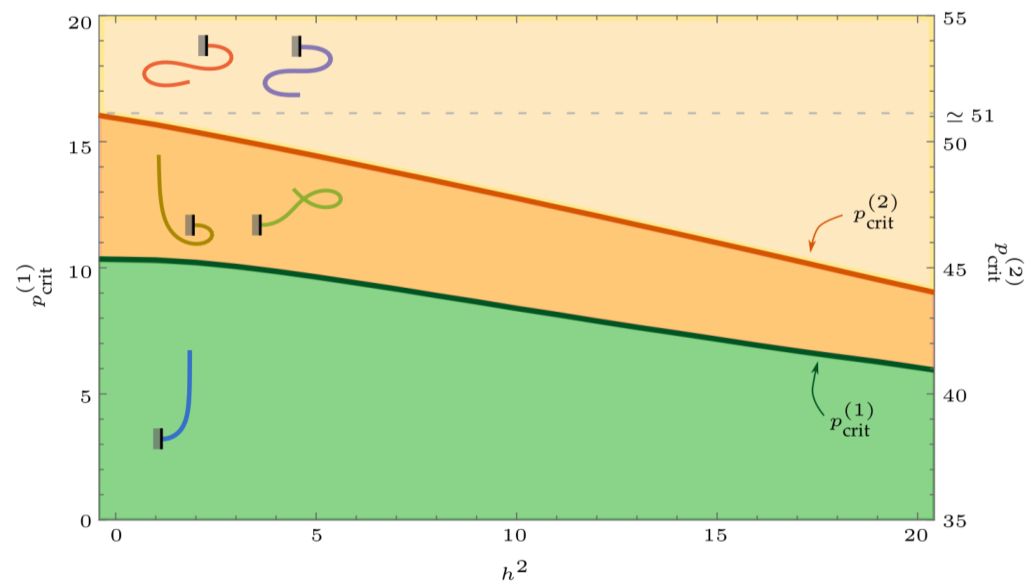

2.0
1.5
1.0
0.5
0
h
0
5
10
15
20
25
β (°)
A
B
C
A
B
C



X_2
h_a
P
φ
X_1
0
ℓ



p
C
B
$\bar{p}$
A
$p^{(1)}_{\rm crit} \simeq 10.33$
β_0
$-\frac{3}{2}\pi$
$\beta^{(1)}_{\rm crit}$
$-\pi$
0
$\frac{1}{2}\pi$



20
15
10
5
0
$p^{(1)}_{\rm crit}$
55
$\simeq 51$
50
45
40
35
$p^{(2)}_{\rm crit}$
$p^{(2)}_{\rm crit}$
$p^{(1)}_{\rm crit}$
0
5
10
15
20
h^2

Summary

- The dispersion of hard magnetic inclusions into a soft matrix is a simple technique to produce soft, remotely controlled actuators that can bear large deformations.
- For prolate, weakly magnetised, sparsely dispersed inclusions, magnetic effect can be accounted for by a *reduced* energy functional that depends only on the deformation.
- Based upon this result, we have derived the governing equations for the quasi-static motion of a rod-like actuator accounting for large rotations/displacements.
- From our case studies, different kind of instabilities emerge, which can be hindered to exploit novel actuator configurations.

Some references

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Books

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